

## i-Notation

### Joshua A. Lewis

A spatial scene is a formal representation of such a description. Spatial objects and their relations are abstracted from their real world counterparts in such a way as to capture the essential features and semantics of the original scene. Being able to take the description of a geographic reality and represent it as a spatial scene is a powerful tool that allows us to reason about the world around us and make meaningful inferences based on the abstractions that we have chosen to represent.

Oftentimes we choose to depict objects such as buildings, roads, and cities as regions, lines, and points—respectively. These objects are essential to how we perceive our world, especially relating to travel and interpreting or generating directions. Much work has been done relating points, lines, and regions to each other, including the 9-Intersection model 9, and RCC-8. These tools allow the relations between binary sets of objects to be depicted, such as *Disjoint*, *Overlap*, and *Meet*. These approaches, and others like them, have also been expanded to include relations with holes and other more finely grained features.

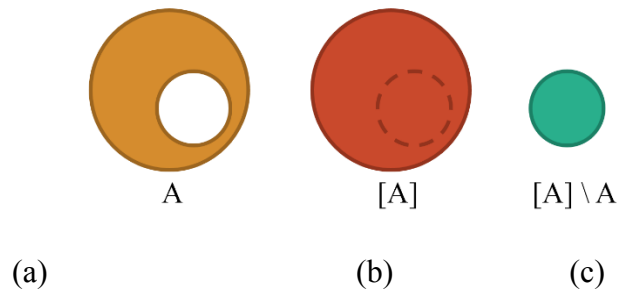
While reasoning with regions is arguably more commonplace, since all objects in our world have extent, traditionally, we still choose to represent many features as lines, such as road networks, pipes, and waterways. Lines have been dealt with exhaustively, with the binary relations between two lines, or a line and a region, being integrated in spatial databases and reasoning software. Similarly, theories of directed lines and oriented lines have been presented in order to represent additional complexity often found within these features, such as one-way streets, or more generally the directed flow of a network. However, by abstracting real objects as lines, there will always be information that is not preserved.

This work aims to represent linear features more robustly as regions, while preserving qualities such as orientation and connectivity. To this end, Hull+o, a descriptive tool for describing spatial scenes (\*\*\***The Topology of Spatial Scenes in R2**, Joshua A. Lewis, Matthew P. Dube, Max J. Egenhofer, 2013\*\*\*), is further developed for this purpose.

## Hull

$Hull+o$  accounts for the placement of objects within or outside of a collection of objects, and accounts for the boundary sequence interactions of those objects, in order to capture a spatial scene completely. The two components of  $Hull+o$ , the *topological hull* and *o-notation*, will be introduced briefly in order to motivate further developments.

**Definition 1:** Let  $A$  be a closed, path-connected set in  $\mathbb{R}^2$  with co-dimension 0 within the standard topology, and let  $B$  be the smallest closed set homeomorphic to an  $n$ -disk such that  $A \subseteq B$ .  $B$  is called the *topological hull of  $A$* , denoted as  $[A]$ .  $\circ$



**Fig. 3:** (a) A closed disk  $A$ , with a path-connected interior in  $\mathbb{R}^2$ , (b) the object's topological hull, such that  $\delta A \subseteq [A]$ , (c) the hole of  $A$ ,  $[A] \setminus A$ .

To represent relations with holes, those holes need to be accounted for (Fig. 3). To this end the *topological hull* acts as a hole-filling operation, allowing objects representative of any holes within an object to be defined by subtracting the original object from its hull.

## Aggregate Hull

**Definition 2:** Let  $A$  be a closed set in  $\mathbb{R}^2$  with co-dimension 0 within the standard topology. Consider the collection of path-connected subsets  $P$  such that  $\bigcup_{p \in P} p = A^\circ$  and  $|P|$  is minimized. The *aggregate topological hull*  $[A]$  is the set  $B = \bigcup_{p \in P} [p]$ .  $\circ$

The topological hull is a special case of the aggregate topological hull, and maintains the same notation. As the aggregate topological hull of an object  $A$  is the union of  $A$ 's path-connected subsets, the aggregate topological hull allows separations—or satellites—of objects to be identified, in addition to any holes within each subset.

## **o-Notation**

The second contribution of *Hull+o*, *o-notation*, provides an accounting of all of the boundary sequence interactions within a scene. There are two components to boundary interactions—dimension and sequence. The *o-notation* distinguishes between both 0 and 1 dimensional touching relations, as well as between 0 and 1 dimensional crossing relations between scenes of objects. The sequence of interactions is also captured, by traversing the boundary of each object and recording each interaction in order. Together, the dimension and sequence information are recorded using the following notation:

$$\partial A_{comp}: o_S(\textit{dimension}, T, C)$$

Symbol  $A_{comp}$  is a boundary component of a region  $A$ ,  $S$  is the collection of regions of which that boundary component is currently *outside* (symbolized by the letter  $o$ ), *dimension* is the qualitative length of the interaction (either 0 or 1),  $T$  is the collection of region boundaries that are undergoing a *touch* interaction, and  $C$  is the collection of regions undergoing a *cross* interaction.

## **i-Notation**

While *o-Notation* delivers a powerful means for describing the boundary interactions between multiple objects, it suffers from a degree of redundancy. Namely, when considering a scene with many objects it is often the case that most *o-Notation* strings will have large and similar sets of objects listed in the *outside* component of the string. To remedy, consider the following alteration:

$$\partial A_{comp}: i_S(\textit{dimension}, T, C)$$

Here  $S$  is the collection of regions for which the current boundary intersection is currently *inside* of, symbolized by the letter  $i$ . The other components remain unchanged. This modified approach will be referred to as *i-Notation*. With this seemingly minor modification, we only record those objects that a boundary exists inside of for some duration, such as during an *overlap*, *coveredBy*,

*inside*, or *equals* relation. Instead of potentially recording all objects within a scene with o-Notation, such as in *disjoint*, we now only record information that immediately pertains to the target object—objects with a shared interior.

Using i-Notation it is possible to recreate all 8 region-region relations described by the 9-Intersection in R2.

<b>Binary Relation:</b>	<b><math>\partial A</math></b>	<b><math>\partial B</math></b>
Disjoint	$i_{\{\emptyset\}}( )$	$i_{\{\emptyset\}}( )$
Meet	$i_{\{\emptyset\}}(*, B, \emptyset)$	$i_{\{\emptyset\}}(*, A, \emptyset)$
Overlap	$i_{\{\emptyset\}}(*, \emptyset, B) i_{\{B\}}(*, \emptyset, B)$	$i_{\{\emptyset\}}(*, \emptyset, A) i_{\{A\}}(*, \emptyset, A)$
Equal	$i_{\{B\}}(1, B, \emptyset)$	$i_{\{A\}}(1, A, \emptyset)$
CoveredBy	$i_{\{B\}}(*, B, \emptyset)$	$i_{\{\emptyset\}}(*, A, \emptyset)$
Inside	$i_{\{B\}}( )$	$i_{\{\emptyset\}}( )$
Covers+	$i_{\{\emptyset\}}(*, B, \emptyset)$	$i_{\{A\}}(*, A, \emptyset)$
Contains	$i_{\{\emptyset\}}( )$	$i_{\{A\}}( )$

Certain relations, specifically *Meet*, *Overlap*, *CoveredBy*, and *Covers*, can be part of a sequence of i-Notation strings. For instance, and object A could *Meet* an object B, then *Overlap* it, and then *Meet* it again, leading to a complex depiction of the relation between A and B. This is called Stacking, since we can stack specific i-notation string together, to create a more robust depiction than is usually possible.

### **i-Notation (lines as regions, orientation)**

While the 8 region-region relations translate directly to i-Notation, the 19 region-line relations and 33 line-line relations require more interpretation as lines are not compatible with the topological hull, which requires a closed, path-connected boundary. However, representing lines as regions not only prevents the invalidation of the theory, it also allows linear features to take on a more robust representation than is typically possible.

Orientation is one of the attributes this approach allows us to explore. Each day we drive about our roadways on a specific side, we rotate a screw one way to loosen it, and another to tighten, and we race about a track in an established direction (hopefully). This quality is not restricted to human-made features, however. Wind flows about a wing in a specific fashion, water can swirl about in a persistent whirlpool in the middle of a river, and planetary bodies spin about their axes.

In order to accommodate orientation, the i-Notation presented earlier will be further modified:

$$\partial A_{comp}: i_S(\textit{dimension}, T, C, \mathbf{O})$$

Here the symbol O represents orientation of the object. Each intersection then depicts the orientation between itself and the next intersection in the *i-Notation* sequence. Orientation can take on four specific values; 0 for *unoriented*, 1 for *bidirectional*, 2 for *clockwise*, and 3 for *anticlockwise*. With these simple additions it is possible to start modeling features such as traditional lines, one-way and two-way roads.

### **i-Notation (traversability)**

The final addition is the incorporation of traversability—identifying the valid connections within a network of objects. To consider this, imagine two roadway components you might encounter each day: a 4-way intersection and an over/underpass. It makes sense, from the standpoint of a driver, to be able to travel from one road to another at the 4-way intersection, by making a left or a right turn. However, such a transition is not desirable when traveling on an overpass.

Since the objects we're dealing with are embedded in the plane, distinctions such as these, which are very dependent on eight, are not generally possible. To vanilla i-Notation these both look the same. We need to represent explicitly when we can make such transitions within a network.

Thus, final piece of the modified i-Notation is as follows:

$$\partial A_{comp}: i_S(\textit{dimension}, T, C, O, \mathbf{R})$$

Here  $R$  is the set of objects that can be reached from a given intersection. The value of  $R$  is relative to the object currently being traversed—this is not necessarily shared by all paths involved within that intersection. For instance, if we can reach roads  $B$  and  $C$  at an intersection, the value of  $R$  would be the set  $\{B,C\}$ .

## **Conclusions/Future Work**

To conclude, I-Notation allows the representation of spatial scenes, and can be expanded to include features such as the orientation of regions, and the connectivity of networks. i-Notation allows many objects to be handled at once, a features not present in much contemporary work. Furthermore, with the addition of the topological hull, holes within objects, and ensembles of objects that surround one another can be depicted.

In the future, Hull+I will be normalized, to remove some of the redundancy in the representation. Another goal is to use hull+I to develop an automatic means of generating a spatial scene from a scene description.