Qualitative Topological Determination from Metric Information

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Abstract

In geographic information science, a set of eight measures have been introduced to offer metric refinements of topological relations between two regions. Examples include the percentage of the common interior of two overlapping regions and the relative increase to convert to disjoint regions into touching regions. Some of these measures apply only to particular relations yielding a functional dependency from the metric measures to the topological relations. The purpose of this paper is to establish the inherent linkages between the eight qualitative region-region relations and the eight quantitative metric values by comparing their impacts upon the qualitative 9-intersection matrix. We prove that from the value ranges of the metric measures one can infer uniquely the topological relation. We provide a comprehensive account of all 432 mappings and give visual examples of the 17 viable options, each producing a unique relation. Comparing these results, minimal conditions are set forth for region-region relation determination. The impact of this new insight on future geographic information systems includes the ability to check topology without ever drawing the objects and the ability to convert numerical values into polygonal entities.

1. Introduction

This paper is concerned with spatial objects that have a continuous boundary and have no holes. All of these objects are located in a simple two-dimensional plane. Something like: such spatial objects are called *spatial relations*. We consider in all instances two objects, one named A and one named B, that have an unknown topological relation between them that we wish to establish. Every object we consider has three distinct features: (1) an interior (denoted as A^o), (2) a boundary (denoted as ∂A), and (3) an exterior (denoted as A'). Interior is defined as … **and the other two as well.

Start reversely: The 9-intersection considers the interplay of two regions' interiors, boundaries, and exteriors by analyzing the intersections of these six parts. This leads to a 3x3 matrix, the 9-intersection matrix, in which typically empty and non-empty intersections are registered. As such the 9-intersection matrix offers a qualitative description for the regions' binary relation as no further details are captured for non-empty intersections. The empty and non-empty intersections distinguish uniquely eight topological relations between two spatial regions (disjoint … equal). This set of relations is jointly exhaustive and pairwise disjoint (i.e., ***).

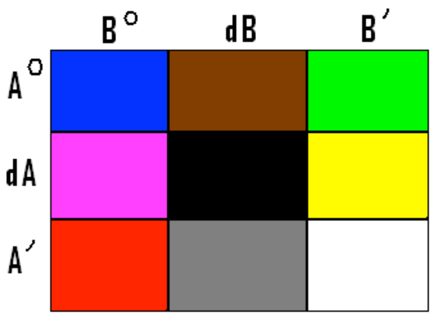

Figure 1: 9-Intersection Matrix

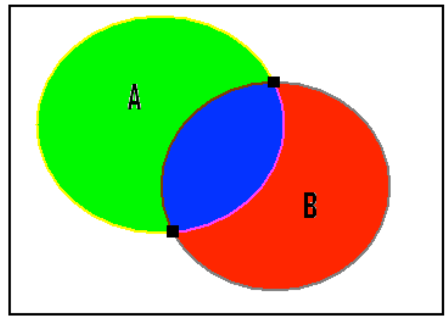

Figure 2: Object Form Representation

Create a single table that contains the eight matrices together with their example drawings.

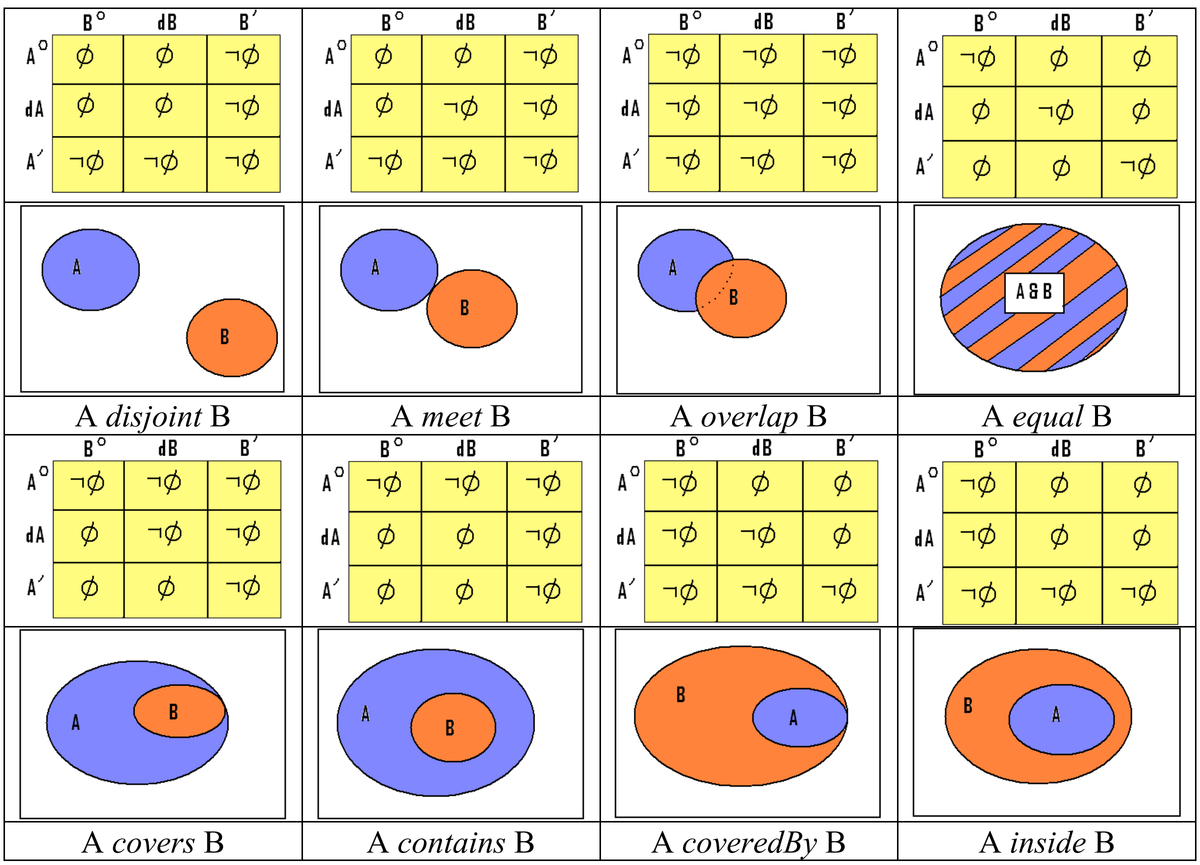

Figure 3: The eight topological relations between two regions and their corresponding 9-intersection matrices.

Given the representations, we can now distinguish the measurable quantities.

1. Inner Area Splitting (portion of A inside of B) (0<=x<=1)
2. Outer Area Splitting (portion of A outside of B) (0<=x<=1)
3. Exterior Splitting (area of exterior shut off by the union of A and B) (0<=x)
4. Inner Boundary Traversal (portion of A's boundary inside of B) (0<=x<=1)

5. Outer Boundary Traversal (portion of A's boundary outside of B) (0<=x<=1)
6. Boundary Sharing (portion of A's boundary shared with B) (0<=x<=1)
7. Inner Closeness (contraction required for A and B to touch boundaries) (1<=x or N/A)
8. Outer Closeness (swelling required for A and B to touch boundaries) (1<=x or N/A)

Figures 11-18 highlight the measures of interest in red. Same with these figures: create a single table. Figures look rudimentary. You can work off the attached figures. I guess you'll need some practice with a slightly more advanced drawing program.

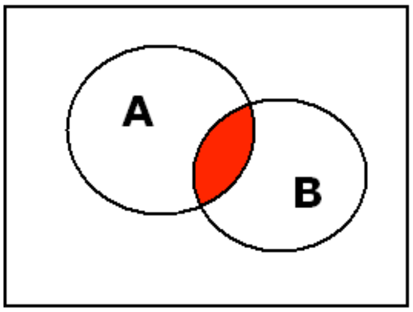

Figure 11: Inner Area Splitting

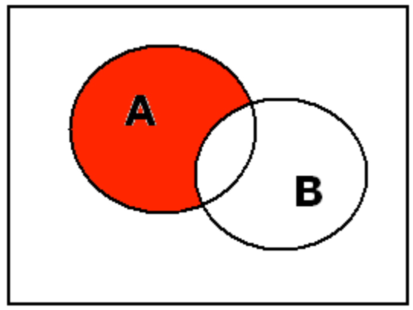

Figure 12: Outer Area Splitting

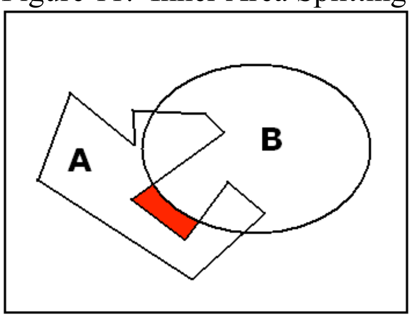

Figure 13: Exterior Splitting

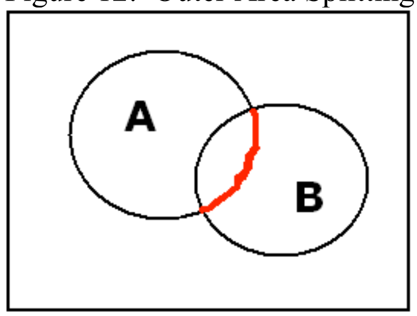

Figure 14: Inner Boundary Traversal

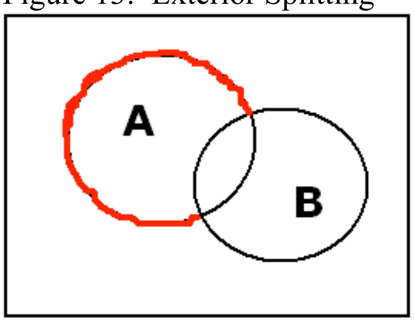

Figure 15: Outer Boundary Traversal

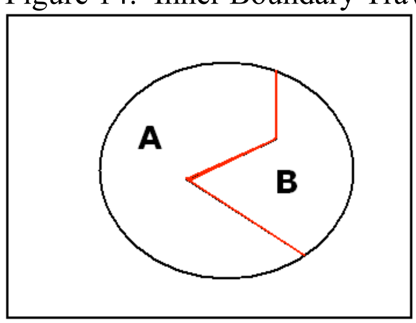

Figure 16: Boundary Sharing

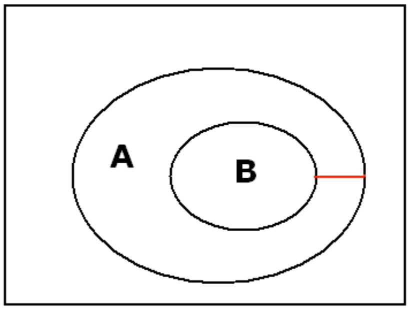

Figure 17: Inner Closeness

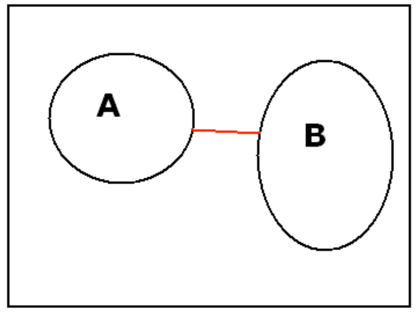

Figure 18: Outer Closeness

Here starts probably

FINDINGS. Starting with the definitions of the quantitative metrics, we can isolate cells of the 9-intersection matrix that would be affected by the measure. Using this information, we can then proceed to isolate topological information from the metrics.

Inner Area Splitting:

$\text{Area}(A^o \cap B^o)/\text{Area}(A)$

Term "Cell" has not been defined.
Need to write the equations with the Word's equation editor (make sure that same font size is used as for text)

By the definition of inner area splitting, it is clear that the value of inner area splitting will be crucial in defining Cell A in the 9-intersection matrix, just as Cell A will be a driving force in the value of the inner area splitting metric. It is clear that if Cell A is empty, that means that the inner area splitting metric must equal 0, since $\text{Area}(A^o \cap B^o) = 0$ when Cell A is empty. Of our topological combinations, only *disjoint* and *meet* satisfy that Cell A is empty, thus *disjoint* and *meet* must have inner area splitting = 0. No other topological configuration can have inner area splitting = 0 since the rest have finite intersections in Cell A.

If Cell A is non-empty, we must check other components. Cell A's status as non-empty eliminates *disjoint* and *meets*.

If inner area splitting = 1, all of A's interior is in B's interior since $\text{Area}(A^o \cap B^o) = \text{Area}(A)$, therefore Cell B and Cell C must be empty (Cells A – C denote the presence of A's interior in relation to B's interior, boundary, and exterior respectively). This condition is fulfilled by *equals*, *covered by*, and *inside*.

If inner area splitting = (0,1), some of A's interior is in B's exterior, B's boundary, and B's interior. It is impossible for the interior of A to be in any two of these without the third due to completeness of the real line. Therefore Cell A is non-empty, Cell B is non-

empty, and Cell C is non-empty. This condition is satisfied by *overlaps*, *covers*, and *contains*.
Recapping all of this, the value of inner area splitting uniquely defines the configuration of the first row in the 9-intersection matrix.

Outer Area Splitting

Area($A^o \cap B'$)/Area(A)

Outer Area Splitting is the direct inverse of Inner Area Splitting. All relations remain the same, all that is required is to subtract each value from 1, as outer area splitting provides the complementary set to the inner area splitting because B^o and B' are complements.

Exterior Splitting

Area(enclosed region)/Area(A)

Exterior Splitting is the most unique of the measures that we have. Exterior Splitting has the potential to be created when we have at least two separate intersections between ∂A and ∂B. The matrix itself has no way of distinguishing between the cardinality of an intersection, but it does contain information vital to exterior splitting in the form of the intersections that must happen minimally.

For exterior splitting to occur, we must have an intersection between ∂A and ∂B. This means that Cell E must be non-empty. Furthermore, both A' and B' must intersect, meaning that Cell I is non-empty. Both ∂A and ∂B must intersect the opposite exterior as well, meaning that both Cell F and Cell H are non-empty. The interiors must also intersect the opposite exteriors since part of each object must be outside the other to create exterior splitting. This means that Cell C and Cell G are non-empty.

Cell A, Cell B, and Cell D remain as variable so far. Searching the matrices for the above conditions, we find that when exterior splitting exists, we only have two possible outcomes: *overlaps* and *meets*.

If exterior splitting does not occur, then the value for it is obviously 0. For this to happen, Cell I must be non-empty. In the given situation of a two-dimensional flat plane, this is inconsequential because Cell I must always be non-empty.

Inner Traversal Splitting/Outer Traversal Splitting/Boundary Sharing

These three metrics refer to the distribution of the boundary of A. Adding the three together must result in 1. This is important to remember later. Given that this is a matter of the distribution of A's boundary, we are concerned with the values contained in Row 2 of the 9-intersection matrix. Some are valid; some are not. Table 1 shows the specific entries that could happen in Row 2 of this matrix and addresses why or why not they are possible.

Table 1: Boundary Row Possibilities

$\partial A \cap B^{o}$	$\partial A \cap \partial B$	$\partial A \cap B'$	Ramification	Possibility
empty	empty	empty	A has no boundary	impossible
non-empty	empty	empty	A’s boundary in B’s interior completely	possible
empty	non-empty	empty	A’s boundary matches B’s boundary	possible
empty	empty	non-empty	A’s boundary in B’s exterior completely	possible
non-empty	non-empty	empty	A’s boundary in B’s closure	possible
non-empty	empty	non-empty	A’s boundary is both inside and outside but not coincident with B’s boundary	impossible
empty	non-empty	non-empty	A’s boundary not inside B’s interior	possible
non-empty	non-empty	non-empty	A’s boundary in all parts of B	possible

As we have shown, we have six possible middle row configurations. The six configurations have specific topological relations paired with them from the 9-intersection matrix. These results are shown in Table 2:

Table 2: Boundary Possibilities with Topological Relations

Option #	Possible Topology
1	inside
2	equals
3	disjoint contains
4	coveredBy
5	meets covers
6	overlap

By this table we have four relations already functionally dependent. *Inside, equals, covered by,* and *overlap* have unique configurations on the center row.

Let's look at these four closer. For option 1, we know that A's entire boundary is in B. So inner traversal splitting = 1, outer traversal splitting = 0, and boundary sharing = 0. Let's consider option 4. We know that A's boundary is completely within the closure of B. It is possible for ∂A and ∂B to meet in multiple places with no length, namely points. Therefore inner traversal splitting = 1, outer traversal splitting = 0, and boundary sharing = 0. We have two configurations that are now not unique. Table 3 shows the range of values acceptable for the measures in each option (note that options 3 and 5 will not have unique representations for their parts):

Table 3: Range of Values for Boundary Splits

Option	Inner	Outer	Sharing
1	1	0	0
2	0	0	1
3	0	1	0
4	0<x<=1	0	0<=x<1
5	0	0<x<=1	0<=x<1
6	0<x<1	0<x<1	0<=x<1

As one can see, it is easy to have values that would look the same for different combinations. The only unique definition here is *equals* (option 2).

Inner Closeness

Width of buffer/Area(A)

To observe inner closeness, otherwise being referred to as contraction closeness, A must be at least as large as B and B must not extend past A's boundary. This means by shear verbal standards, that there are three options: *equals*, *covers*, and *contains*.

There is proof of this through the 9-intersection matrix. We start with the notion that B cannot extend past A's boundary. By virtue of completeness of the real line, we know that A^o and B^o must intersect, yet B^o and ∂A do not intersect. This establishes fully Column 1 of the 9-intersection matrix. To register a value of inner closeness, Cell A must be non-empty, while Cell D and Cell G are both empty. The three satisfactory models are thus *equals*, *covers*, and *contains*.

Can we refine this further? The answer is yes.

For inner closeness to reach a value of 0, we know through virtue of the completeness of the real line that ∂A and ∂B must intersect, meaning that A need not shrink to create boundary contact. This condition fills in Cell E in the 9-intersection matrix. We have thus made the following refinement: if inner closeness>0, then we have *contains*. If inner closeness = 0, we have either *equals* or *covers*.

Outer Closeness

(Area(A) + swelling distance)/Area(A)

Outer closeness is a hybrid model. Outer closeness is more aptly described as the amount by which A must swell to contact B's boundary. Intuitively, this seems like A must be outside of B, namely the two objects either *meet* or are *disjoint*. It is completely possible however to conceive of A being smaller than B and needing to swell to contact B's boundary, namely A is *inside*, *equals*, or is *covered by* B.

We must treat the cases separately unfortunately because the defining characteristics of the relations in question are so far opposite. We get the notions of "outside" and "inside" through the value of inner area splitting to make the distinction.

For inner area splitting = 0 (a.k.a. A outside of B), we need to maintain all exterior intersections. Since we know that inner area splitting = 0, we throw away *overlap* and we are left with *meet* and *disjoint*. Cells C, F, G, H, and I have non-empty intersections.

We can go further than this. Since *meet* has an intersection in Cell D, that means that A and B touch. This means that outer closeness = 1 because the swelling distance = 0. If A and B were *disjoint*, then there would be some swelling distance, and outer closeness > 1.

For the case where inner area splitting ≠ 0, the closure of A must be completely contained by the closure of B, thus we have empty exterior intersections with the exception of Cell I. We also know that the interiors (Cell A) must intersect because of the closure containment. These distinctions result in *equals*, *covered by*, and *inside*.

We can refine this further. If outer closeness > 1, then Cell E is empty. If Cell E is empty out of these options, we know that A must be *inside* B. If outer closeness = 1, then we are left with *covered by* and *equals*.

<u>The Sets</u>

Given the constructs above, we have the following options to work with for values:

3 possible sets for inner area splitting/outer area splitting
2 possible sets for external splitting
6 possible sets for inner boundary traversal/outer boundary traversal/boundary sharing
3 possible sets for inner closeness
3 possible sets for outer closeness

Given these values, there are 324 possible combinations of the metrics. This assumes that they can all happen with each other. This assumption is of course faulty. As we went through the different metrics, we established which of the relations would satisfy the conditions set. Since the topological relations are mutually exclusive and jointly exhaustive, we can easily reduce this number in a systematic way by starting with the inner area/outer area options.

Assume that we have observed an inner area splitting of 0. This means we have one of two cases: *disjoint* or *meet*. Of the two possible sets for external splitting, both contain either *disjoint or meet*. Of the six possible sets for the boundaries, two of them contain *disjoint or meet*. Of the three possible sets for inner closeness, only one contains *disjoint* or *meet*. Of the three possible sets for outer closeness, two contain *disjoint* or *meet*. That being said, we have eight possible viable relations from the first condition (2 x 2 x 2).

Assume that we have now observed inner area splitting of 1. This means we have one of three cases: *equals*, *inside*, and *covered by*. Of the two possible sets for external splitting, only one contains these relations. Of the six possible sets for the boundaries, three contain these values. Of the three possible sets for inner closeness, two of them contain these values. Of the three possible sets for outer closeness, two of them contain these values. That being said, we have twelve candidate relations from the second condition (3 x 2 x 2).

Assume now that we have observed inner area splitting of (0,1). This means we have one of three cases: *overlap*, *contains*, and *covers*. Of the two possible sets for external splitting, both contain these relations. Of the six possible sets for the boundaries, three contain these values. Of the three possible sets for inner closeness, all three contain these values. Of the three possible values for outer closeness, only one option contains the relations. That being said, we have eighteen candidate relations from the third condition (2 x 3 x 3). This brings us to a grand total of 38 potentially viable relationships. This represents a reduction of roughly 88%, a very good first step. Not all of these potentially viable relationships are viable however. We need to drill into the combinations to find contradictions between them. Table 4 shows the 38 combinations that are viable based on topological combination and addresses their outcomes/contradictions.

Table 4: The Bullshit Table

IAS	ES	IBT	OBT	BS	IC	OC	Result	Contradiction
0	0	0	(0,1]	[0,1)	NA	1	Meet	#1
0	>0	0	(0,1]	[0,1)	NA	1	Meet	#2
0	0	0	(0,1]	[0,1)	NA	>1	Disjoint	#3
0	>0	0	(0,1]	[0,1)	NA	>1	-none-	ES implies meet whereas OC implies disjoint
0	0	0	1	0	NA	1	Meet	#4
0	>0	0	1	0	NA	1	Meet	#5
0	0	0	1	0	NA	>1	Disjoint	#6
0	>0	0	1	0	NA	>1	-none-	ES implies meet whereas OC implies disjoint
1	0	1	0	0	NA	1	Covered by	#7
1	0	1	0	0	NA	>1	Inside	#8

1	0	0	0	1	NA	1	-none-	IC implies not equal, BS implies equals
1	0	0	0	1	NA	>1	-none-	BS implies equals, OC implies separation of boundary
1	0	(0,1]	0	[0,1)	NA	1	Covered by	#9
1	0	(0,1]	0	[0,1)	NA	>1	Inside	#10
1	0	1	0	0	0	1	-none-	Special case of covered by without IC=0
1	0	1	0	0	0	>1	-none-	IC and OC contradict
1	0	0	0	1	0	1	Equals	#11
1	0	0	0	1	0	>1	-none-	Equal boundary but separation in OC
1	0	(0,1]	0	[0,1)	0	1	-none-	Covered by but IC=0 implies A>=B
1	0	(0,1]	0	[0,1)	0	>1	-none-	OC>1 implies inside, most all else lead to covered by
(0,1)	0	0	1	0	NA	NA	-none-	IC and OC together restrict us to overlap, but OBT dictates contains
(0,1)	0	0	(0,1]	[0,1)	NA	NA	-none-	IC and OC dictate overlap, but IBT does not show overlap
(0,1)	0	(0,1)	(0,1)	[0,1)	NA	NA	Overlap	#12
(0,1)	0	0	1	0	0	NA	Covers	#13
(0,1)	0	0	(0,1]	[0,1)	0	NA	Covers	#14
(0,1)	0	(0,1)	(0,1)	[0,1)	0	NA	-none-	Covers with

								a value for OBT
(0,1)	0	0	1	0	>0	NA	Contains	#15
(0,1)	0	0	(0,1]	[0,1)	>0	NA	Contains	#16
(0,1)	0	(0,1)	(0,1)	[0,1)	>0	NA	-none-	Contains with an inside boundary
(0,1)	>0	0	1	0	NA	NA	-none-	ES implies overlap or meet, but IBT=0
(0,1)	>0	0	(0,1]	[0,1)	NA	NA	-none-	ES implies overlap or meet, but IBT=0
(0,1)	>0	(0,1)	(0,1)	[0,1)	NA	NA	Overlap	#17
(0,1)	>0	0	1	0	0	NA	-none-	ES implies overlap or meet, but IBT=0
(0,1)	>0	0	(0,1]	[0,1)	0	NA	-none-	ES implies overlap or meet, but IBT=0
(0,1)	>0	(0,1)	(0,1)	[0,1)	0	NA	-none-	All imply overlap except for IC=0
(0,1)	>0	0	1	0	>0	NA	-none-	ES implies overlap or meet, but IBT=0
(0,1)	>0	0	(0,1]	[0,1)	>0	NA	-none-	ES implies overlap or meet, but IBT=0
(0,1)	>0	(0,1)	(0,1)	[0,1)	>0	NA	-none-	All imply overlap except IC>0

Table 4 shows us that we have seventeen possible combinations that are acceptable topologically, a total reduction of 95%. Table 5 shows the distribution of the topological relations over these seventeen.

Table 5: Topological Distribution

Topological Relation	Number of Instances
disjoint	2
meet	4
overlap	2
equal	1
covers	2
contains	2
coveredBy	2
inside	2

We have now constructed the total sets that define topology, but are there subsets that uniquely define topological relations? The answer to this is yes. For this section, we will assume that the objects we are getting values from actually can be realized, therefore we cannot get wrong values.

<u>Minimal Subsets of Definition</u>

Disjoint

Two definitions resulted in *disjoint*. Both lines have the following values in common: IAS = 0, ES = 0, IBT = 0, IC = NA, and OC > 1. This subset obviously would trivially define *disjoint*. The issue at hand is to distinguish *disjoint* from *meet*, the two exterior relations. The only concrete differentiation between *disjoint* and *meet* is OC > 1. For two objects to meet, their boundaries must coincide at at least one point. This means that meet requires OC = 1. This states that the minimal definition for *disjoint* is IAS = 0 and OC > 1.

Meet

The minimal definition for *meet* is IAS = 0 and OC = 1. There are four ways to obtain *meet*, so it makes sense to see if there is something else hiding in the definitions.

Recall that all rows resulting in *disjoint* contained ES = 0. Two rows resulting in *meet* contained ES > 0. That being said, IAS = 0 and ES > 0 define *meet* uniquely.

A further difference between *meet* and *disjoint* is the ability of two meeting objects to share boundaries. We have a measure for that in BS. IAS = 0 and BS > 0 implies *meet* directly.

Recall from the discussion of exterior splitting that two topological relations could create exterior splitting: *meet* and *overlap*. Both rows resulting in *overlap* contain IBT = (0,1), OBT = (0,1), BS = [0,1), IC = NA and OC = NA. That being said, we have some more unique definitions. *Meet* can be distinguished from *overlap* on the basis of ES > 0 and IBT = 0, and can also be distinguished from *overlap* on the basis of ES > 0 and OC = 1.

Overlap

As one might expect, the first definition of *overlap* is that which distinguishes it from *meet*. *Overlap* is distinguished by ES > 0 and IBT > 0. *Overlap* is also distinguished by ES > 0 and OC = NA. *Overlap* can also be distinguished by IAS = (0,1) and ES > 0.

There were six rows in Table 4 that contained IC = NA and OC = NA. The only result that came of these rows was *overlap*. These are obviously tied together as this means that the object is both inside and outside the object at the same time. This statement leads us to another distinction: *overlap* is the only topological relation that allows for its boundary to be both inside and outside of the related object. That being said, IBT > 0 and OBT > 0 must define *overlap*.

Overlap can also be differentiated from *covers* and *contains* by IC = NA and IAS = (0,1).

Equals

Equals is the trivial case of topology. It requires the object to have the same boundary, the same interior, and the same exterior. These statements lead to specifics that only *equals* can provide together. In our case, it means we have unique definitions. It turns out, *equals* is the only unique definition, meaning there is only one way to get it, namely IAS = 1, ES = 0, IBT = 0, OBT = 0, BS = 1, IC = 0, OC = 1.

The first minimal condition for *equals* is found in BS. BS = 1 means that the entire boundary is shared between the two objects. This means the two objects are *equal* simply because we have simply connected regions. This is the smallest minimal condition that we will find for any topological relation. Trivially, this also means that IBT = 0 and OBT = 0 also defines *equals* because the three together must sum to 1.

The next minimal condition that is intuitively obvious is that *equals* is the only relation that would exhibit both IC and OC values. Both of these metrics are defined in only *equals*. Therefore IC = 0 and OC = 1 uniquely defines *equals*.

In our method of constructing realizable relations, *equals* was tied to both *inside* and *covered by*, all of which have IAS = 1. Is *equals* distinguishable from these? It turns out that it is. *Equals* can be distinguished from both *inside* and *covered by* in that it has IC = 0, therefore IAS = 1 and IC = 0 defines *equals*.

Inside

Inside implies that there is no boundary intersection and that the closure of A is contained within B. This means that IAS = 1 and that BS = 0. Unfortunately, this does not distinguish it from *covered by* as a single point intersection has no length. We must look deeper than this definition.

Given the information about the closure, we can infer that *inside* must exhibit OC > 1. The object must grow to intersect boundaries. Neither *covered by* nor *equals* can have

OC > 1 because they have boundary contact already. The only relation that can have OC > 1 other than *inside* is *disjoint*. The difference between *inside* and *disjoint* is that *inside* has IAS = 1 and IBT = 1, whereas *disjoint* has IAS = 0 and IBT = 0. That said, *inside* has two minimal definitions to distinguish it from *disjoint*, *covered by*, and *equals*: IAS = 1 and OC > 1 and IBT = 1 and OC > 1.

Covered by

Covered by is tied to *meet* and to both *equals* and *inside*. It is tied to both *equals* and *meet* through OC = 1 and is tied to both *equals* and *inside* by IAS = 1.

Covered by appears as though it needs three metrics to define it uniquely. It needs IAS = 1, OC = 1, and IBT $\neq$ 0. This distinguishes it from all three of its competitors. Can we do better? The answer is yes.

The difference between *covered by*, *inside*, and *equals* is the distribution of the boundary. *Covered by* shares some proper subset of the boundary. *Inside* shares no part of the boundary. *Equals* shares the whole boundary. That being said, if 0 < BS < 1, combined with IAS = 1, we must have *covered by*. What this shows is that we get similar information from IC, OC, and BS. The difference between *inside* and *covered by* (as well as the difference between *disjoint* and *meet* and the difference between *covers* and *contains*) shows the necessity of having IC and OC alongside BS. Without IC and OC, we could easily confuse these relational pairs because the boundaries could share only points, which means 0 distance is logged in the metric.

Contains

Contains is tied to both *overlap* and *covers* through IAS = (0,1). We must find some easy way to distinguish it from its partners.

The difference between *contains* and both *overlap* and *covers* is the issue of boundary intersection. *Contains* cannot intersect the boundary, therefore we know that BS = 0. Unfortunately, both *covers* and *overlap* have cases where that also happens, so this information is not enough.

From this information we can derive that there is a separation between the boundaries, namely that IC > 0. Neither *covers* nor *overlap* can meet this criterion. Therefore the minimal definition of *contains* is IAS = (0,1) and IC > 0.

Covers

Covers is different from *contains* in that it has some essence of boundary sharing. *Covers* is different from *overlap* in that A must cover all of B instead of just part of it.

Covers is thus differentiated from *contains* by saying that IAS = (0,1) and IC = 0. *Covers* is thus differentiated from *overlap* in the same way as IC = NA for *overlap.*

Covers cannot be differentiated from *overlap* in regard to boundary sharing. Both *covers* and *overlap* can have intersection of boundary that has finite length.

CONCLUSIONS

All told, we have twenty-one total combinations of minimal conditions for topological relations. These results are presented in Table 6. These twenty-one combinations show that the purpose of the paper is possible and that topology is functionally dependent upon metric information. In fact any topological relation can be defined within at most three metrics (the case when *covers*, *meet*, or *covered by* share only points with the other object). Provided there is finite boundary sharing length, only two are necessary for unique definition.

Table 6: List of Minimal Conditions

Condition 1	Condition 2	Topology Defined
IAS = 0	OC > 1	*disjoint*
IAS = 0	OC = 1	*meet*
IAS = 0	ES > 0	*meet*
IAS = 0	BS > 0	*meet*
ES > 0	IBT = 0	*meet*
ES > 0	OC = 1	*meet*
ES > 0	IBT > 0	*overlap*
ES > 0	OC = NA	*overlap*
IAS = (0,1)	ES > 0	*overlap*
IC = NA	OC = NA	*overlap*
IBT > 0	OBT > 0	*overlap*
IAS = (0,1)	IC = NA	*overlap*
BS = 1		*equal*
IBT = 0	OBT = 0	*equal*
IC = 0	OC = 1	*equal*
IAS = 1	IC = 0	*equal*
IAS = 1	OC > 1	*inside*
IBT = 1	OC > 1	*inside*
IAS = 1	0<BS<1	*coveredBy*
IAS = (0,1)	IC > 0	*contains*
IAS = (0,1)	IC = 0	*covers*

Topology can map onto these metric combinations so long as the third condition is not needed. This is not a functional dependency from the perspective of topology in that there are multiple ways to get equivalent topological combinations.