

Convex Hull-Based Metric Refinements for Topological Spatial Relations

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ABSTRACT

Metric refinements of qualitative topological relations not only provide more details of distinctions between spatial entities, but also theoretical interests and applications. This paper developed an alternative way of metric refinements for improving the identification of the eight topological relations with the aid of the extension of convex hull concept to defining relations between two spatial regions. We defined some area terms and relative ratio factors for metric refinements that can be successfully and uniquely applied to the eight basic relations.

1. Introduction

Qualitative reasoning provides coarse, intuitive, and flexible ways of determining topological relations between spatial objects, and is one of the basic topics of geographic information system (GIS). The motivation of research in qualitative reasoning is also from its potential applications in other different areas such as robotic navigation, spatial propositional semantics of natural languages, engineering design, and high level vision [1]. Previous research work in spatial reasoning has studied various aspects of space, among which topological relations are especially emphasized. Egenhofer et al. (1991, 1993) developed a point-set-based 9-intersection model [2-4], which derives eight fundamental spatial relations similar to those described in the Region-Connection Calculus (RCC) developed by Randell et al. (1992) [5]. The extensions of 9-intersection model have been developed since then, such as the Dimensionally Extended nine-Intersection Model (DE-9IM) and Calculus-Based Method (CMB) by Clementini et al. [6,

7], which becomes a standard used to describe the spatial relations of two geometrical regions, point-set topology, geospatial topology, and fields related to computer spatial analysis. In addition, Chen et al. (1998, 2001) proposed a Voronoi-based 9-intersection model by replacing the exterior of an entity with its Voronoi region [8, 9], which has to certain degree improvement in solving some practical problems in the original 9-intersection model such as difficulties in distinguishing different disjoint relations and relations between complex entities with holes. However, the purely qualitative representation and reasoning or topology *per se* is usually insufficient to depict the spatial relations between objects in the real world, and we need to consider other metric aspects or details of the spatial relations to make to some degree subtle but very important distinctions [1, 10].

The metric details for refinements of different categories of spatial relations either by the 9-interaction or RCC-8 should provide a more precise measure than topology alone. Research efforts in metric spatial reasoning have been addressed on some aspects of space such as direction and orientation [11-14], distance [15], size [16], and shape [17]. Since a metric space exhibits both metric and topological properties [18], some research approaches or models combining topology and metric aspects such as distance and size have been developed [18-23]. Here we are most interested in how to combine distance or closeness with topology to reason the spatial relations. The problem of most previous work is that reasoning with linear distance alone and without consideration of other aspects especially direction often hardly arrives at a conclusion [11, 18].

In this paper, we first introduced the concept of convex hull to define the merged region of two original spatial objects we are trying to deal with the relations and then the outside and inside delta areas which can easily be used to capture the expansion and contraction closeness.

2. Extension of Convex Hull to Topological Relations

The spatial objects or regions considered in this section are embedded in \mathbb{R}^2 as defined in [18], and they have a continuous boundary, no holes, no spikes, and no cuts. In addition, we use the concept of convex hull or polygon of a set of two-dimensional points [24] for a specific spatial region. Let A and B be two convex hulls representing two spatial regions in the two dimensional plane, denoted as $CH(A)$ and $CH(B)$, and we subsequently determine the convex hull $CH(A, B)$ by merging A and B . The convex polygon $CH(A, B)$ is obtained by tracing the two tangents (upper and down) common to A and B [24] (Figure 2).

The eight fundamental topological relations between two such spatial regions or convex hulls here shown in Figure 2 are based on the 9-intersection model [2, 18]. From Figure 2 we can see that the normal convex polygons $CH(A, B)$ defined as above can be obtained only in the relations of *disjoint*, *meet* and *overlap*. In the relation of *equal*, actually, $CH(A, B) = CH(A) = CH(B)$. In the relations of *coveredBY*, *inside*, *covers*, and *contains*, one region is embedded inside another. In order to use the concept of convex hull and determine the contraction closeness for all these relations (to be discussed in details in section 4) we introduce the concept of “mirror” object or right/left dual object for the embedded object. Let the mirror objects of A and B be A' and B' with dotted boundary, respectively, so the imaginary convex hulls with dotted tangents for these four relations are shown in Figure 2. Among these eight topological relations, the non-empty

intersections including meet, overlap and equal are able to capture metric details, which have been addressed in previous studies [25-27]. However, the empty intersections including *disjoint*, *coveredBy*, *inside*, *covers*, and *contains* are unable to be further distinguished with more metric details.

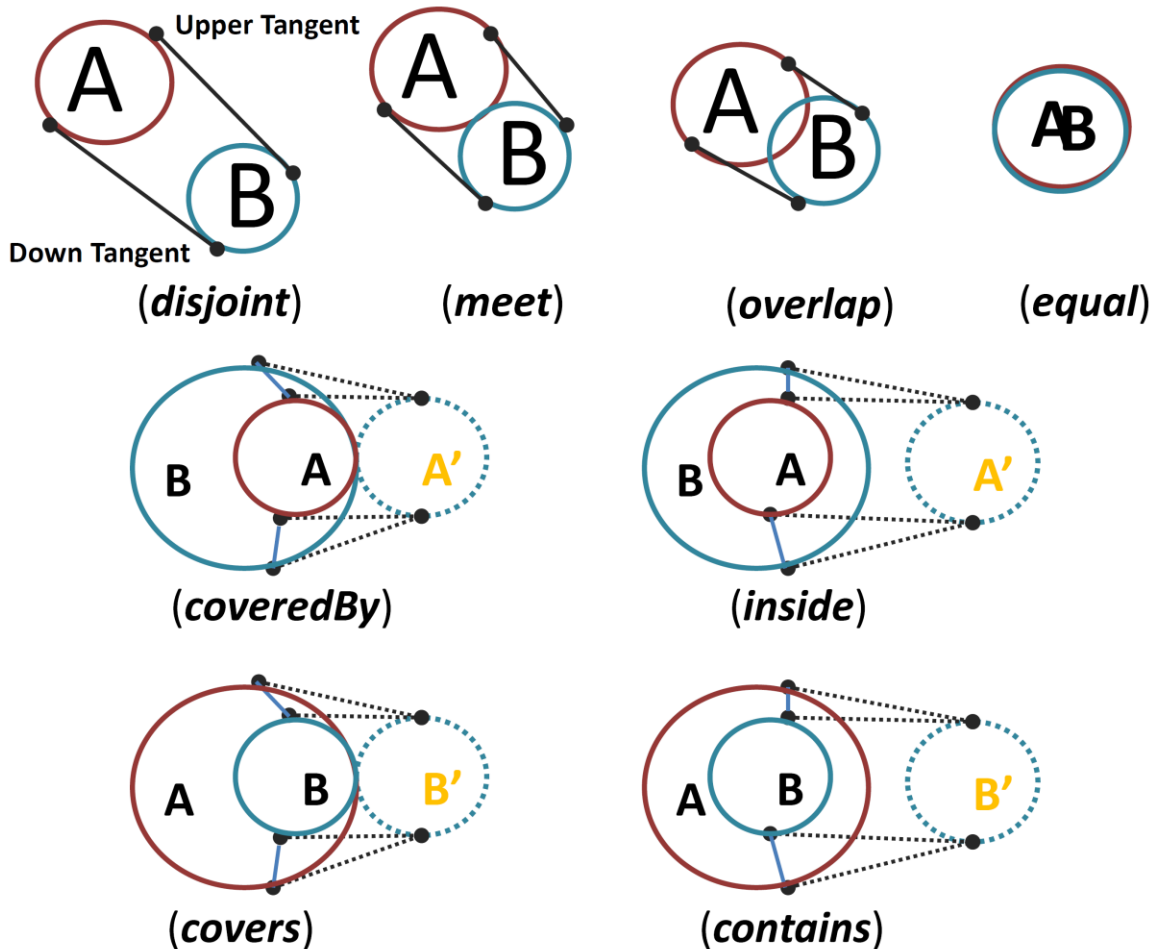


Figure 2. The eight topological relations between two regions in \mathbb{R}^2 with their merged convex hull representations.

3. Convex-Hull-Based Metric Refinements for Region-Region Relations

Metric details are used to refine and enhance qualitative and coarse topological relations. Two types of measures for metric refinements for region-region relations have been developed in [18],

one called *splitting measures* for distinguishing non-empty intersections involving 0, 1, and 2-dimensional intersections, and another called *closeness measures* for refining empty intersections including expansion closeness (EC) and contraction closeness (CC). In this section, we are trying to develop an alternative way to refine region-region relations with the aid of convex hull concepts defined in the previous section. What we especially emphasize here is to use the relative or scale-independent measure of area values to refine either empty or non-empty intersections.

Let us consider three convex hulls here, $CH(A)$, $CH(B)$ and $CH(A, B)$, corresponding to the two original spatial regions we are dealing with what kind of relations will be and the newly emerged reference spatial region. Considering the boundaries of the two original spatial objects A and B in $CH(A, B)$, there is some extra area or space between $CH(A, B)$ and the original $CH(A)$ and $CH(B)$ in the relations of *disjoint*, *meet*, and *overlap* (Figure 2 and 3). Note that we exclude the exceptions as shown in Figure 4 in which no extra area needed to form $CH(A, B)$ from $CH(A)$ and $CH(B)$, i.e, $CH(A, B) = CH(A) + CH(B)$. We define this area as the outside delta area, denoted as Δ_{outside} , and the specific outside delta areas in the relations of *meet*, *disjoint*, and *overlap* as Δ_{meet} , Δ_{disjoint} , and Δ_{overlap} , respectively. We also define the common parts in the overlap relation as Δ_{ab} . While dealing with the relations of *coveredBy*, *inside*, *covers* and *contains* we need to use the mirror or right dual objects of the smaller one as references, which are inside another. By creating the imagined convex hulls, correspondently, we can define the inside delta area, denoted as Δ_{in} , and the specific inside delta areas in the relations of *coveredBy*, *inside*, *covers*, and *contains* as $\Delta_{\text{coveredBy}}$, Δ_{inside} , Δ_{covers} , and Δ_{contains} , respectively. Therefore, no matter what empty or non-empty intersection we deal with, we can define a ratio of the outside or inside delta area with respect to the area of the either of two spatial regions. In the *overlap* relation we also define a ratio of the common part with respect to the area of one of two original regions. We define three major area ratio factors as follows:

- Outside Closeness (OC): the expanding or outside delta area required so that together with the original $CH(A)$ and $CH(B)$ to form $CH(A, B)$. This is similar to the concept of Expansion Closeness (EC) defined in [18] but the difference is that OC does not consider entire buffered zone. OC can be used in the relations of *meet*, *disjoint*, and *overlap*.
- Inside Closeness (IC): the inside delta area proportionally reflecting the imagined outside delta area required so that together with the bigger region and the mirror of the smaller region to form the imagined $CH(A, B)$. This is similar to the Contraction Closeness (CC) in [18] but not using the contracted and buffered delta zone as measure. IC can be applied for metric refinements for the relations of *coveredBy*, *inside*, *covers*, and *contains*.
- Intersection Degree (ID): the common or intersecting portion with respect to the area of $CH(A)$ or $CH(B)$. ID is the same as the inner area splitting (IAS) defined in [18].

Corresponding formula for the ratio factors above and some other related terms are defined below:

$$OC = \frac{\Delta_{\text{outside}}}{\text{Area}(CH(A)) + \Delta_{\text{outside}}}$$

$$IC = \frac{\Delta_{\text{in}}}{\text{Area}(CH(A)) + \Delta_{\text{in}}}$$

$$ID = \frac{\Delta_{\text{ab}}}{\text{Area}(CH(A))}$$

In the relations of meet and disjoint:

$$\Delta_{\text{outside}} = \text{Area}(CH(A, B)) - \text{Area}(CH(A)) - \text{Area}(CH(B))$$

In the relation of overlap:

$$\Delta_{\text{outside}} = \text{Area}(CH(A, B)) - \text{Area}(CH(A)) - \text{Area}(CH(B)) + \Delta_{\text{ab}}$$

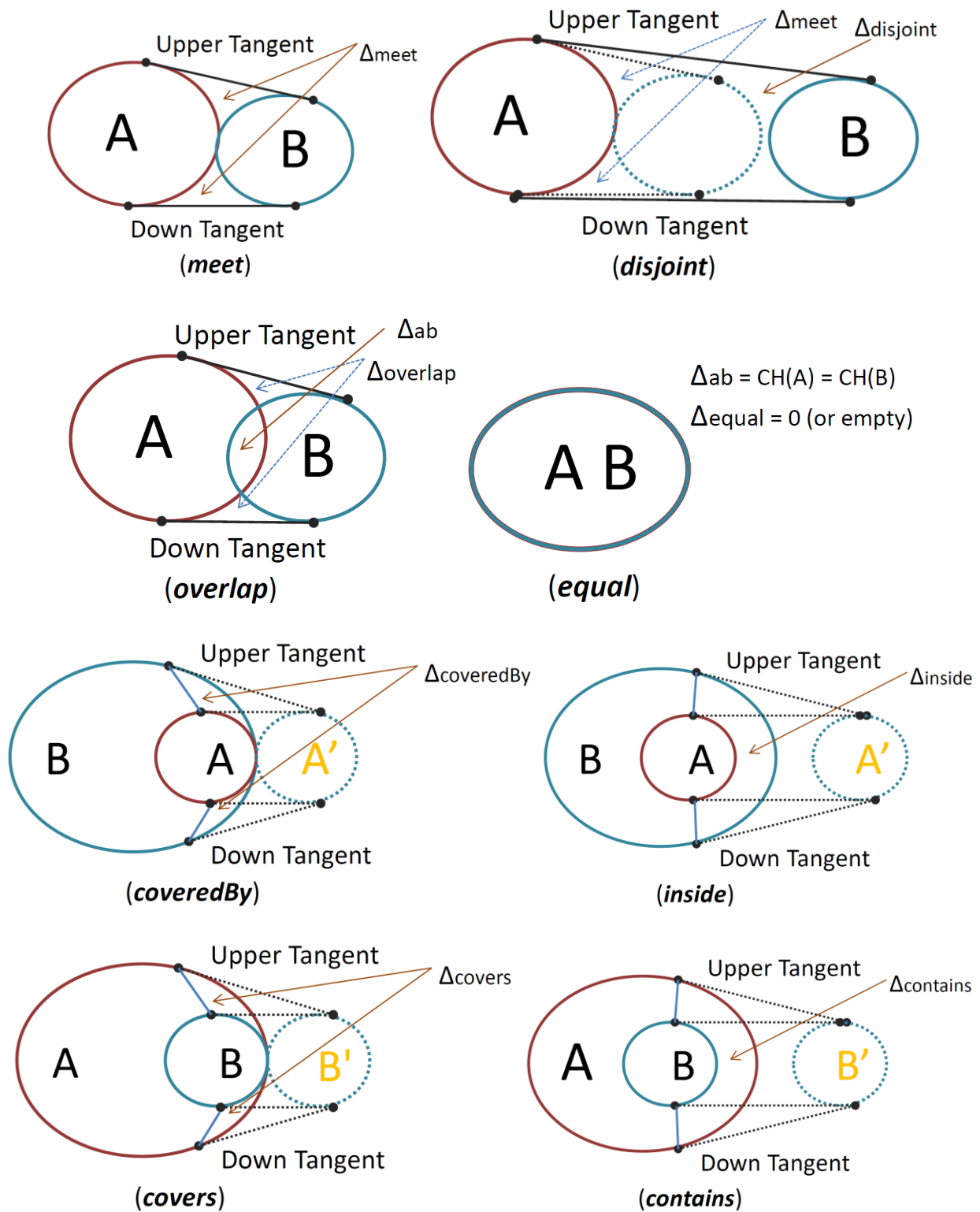


Figure 3. Specification of the common portion (Δ_{ab}), inside and outside delta areas (Δ_{inside} and $\Delta_{outside}$) between two spatial regions A and B for the eight topological relations.

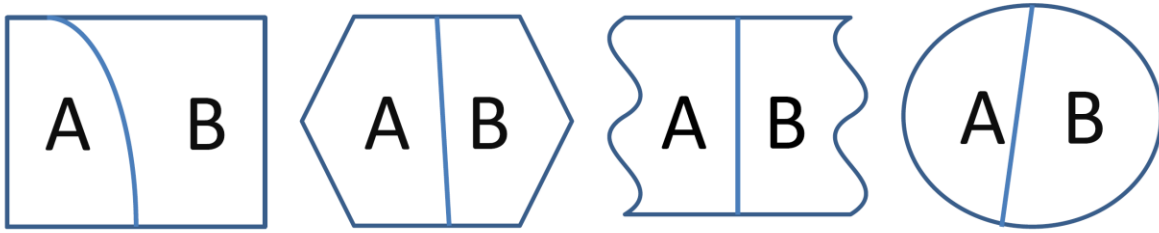


Figure 4. Selected exceptions for $CH(A, B) = CH(A) + CH(B)$ when spatial region A meets region B.

4. Refinement Specifications to Region-Region Topological Relations

In this section, we study the metric refinements for each of eight topological relations with OC, IC, ID and other metric terms defined in this paper. Different metric refinements with different values can be used to identify the specific topological relations.

For the relation of *disjoint* there is a largest Δ_{outside} compared to other relations. The Outside Closeness (OC) for *disjoint* is based on a greater than zero Δ_{outside} , meaning that the value range should be greater than zero and less than one. We can also standardize the OC, denoted as OC', by replacing Δ_{outside} with $\Delta_{\text{disjoint}} = \Delta_{\text{outside}} - \Delta_{\text{meet}}$.

In the relation of *meet* we can define Δ_{meet} as the base outside delta area because we can definitely identify one unique or fixed Δ_{meet} from one direction. So, we consider the OC for *meet* as the base OC (the range value: greater than zero and less 1) for differentiating it from the relations of *disjoint* and *overlap*. We will have $OC_{\text{overlap}} < OC_{\text{meet}} < OC_{\text{disjoint}}$.

In the relation of *overlap* there is a smallest Δ_{outside} , specifically denoted as Δ_{overlap} . Therefore, the OC_{overlap} is the smallest as shown above although the value range is still greater than zero and less than one. However, for the relation of *overlap* we have another ratio factor, Intersection Degree (ID), reflecting the degree of overlapping or common area. The relation between OC_{overlap} and ID is that the less the OC_{overlap} is, the larger the ID.

In the relation of *equal* there is no either Δ_{outside} or Δ_{in} . However, Δ_{overlap} reaches the maximum, i.e. $\Delta_{\text{overlap}} = \text{Area}(CH(A)) = \text{Area}(CH(B)) = \text{Area}(CH(A, B))$. For the equal relation, no expansion and contraction needed, i.e., $OC = 0$ and $IC = 0$.

For the relations of *coveredBy* and *covers* we can define Δ_{in} as $\Delta_{\text{coveredBy}}$ and Δ_{covers} , the base inside delta areas because we can determine one unique either $\Delta_{\text{coveredBy}}$ or Δ_{covers} from one direction inner contact. So, we consider the IC for *coveredBy* as the base IC (the range value: greater than zero and less 1) for differentiating it from the relation of *inside*, and the IC for *covers* as the base IC different from the IC of the *inside* relation. We will have $OC_{\text{coveredBy}} < IC_{\text{inside}}$ and $IC_{\text{covers}} < IC_{\text{contains}}$.

For the relations of *inside* and *contains* we have noticed above that the Δ_{in} is larger than either $\Delta_{\text{coveredBy}}$ or Δ_{covers} because the smaller region is away from the inner boundary. Similarly, the Inside Closeness (IC) for *coveredBy* and *covers* is based on a greater than zero Δ_{in} , implying that

the value range is greater than zero and less than one. We can also standardize the IC, denoted as IC', by replacing Δ_{in} with $\Delta_{inside} = \Delta_{in} - \Delta_{coveredBy}$ or $\Delta_{contains} = \Delta_{in} - \Delta_{covers}$.

While dealing with the eight topological relations between two investigated spatial regions, we can always determine the unique area terms and ratio factors for the relations of *meet*, *coveredBy* and *covers* as the references for comparing the relative expansion and contraction closeness. To show that metric refinements based on these area terms are sufficient to distinguish all the eight fundamental topological relations, we construct the table 1 showing the distinctions between these relations in terms of metric refinements defined in this paper.

Table 1. A table showing different combinations of metric refinements with area terms for distinguishing the eight topological relations.

Topological relation	$\Delta_{outside} / OC$	Δ_{ab}	Δ_{in} / IC
disjoint	$>\Delta_{meet} / > OC_{meet}$	null	null
meet	$\Delta_{meet} / OC_{meet}$	null	null
overlap	$<\Delta_{meet} / < OC_{meet}$	+	null
equal	0	max	0
coveredBy	null	null	$\Delta_{coveredBy} / IC_{coveredBy}$
inside	null	null	$>\Delta_{coveredBy} / IC_{coveredBy}$
covers	null	null	$\Delta_{covers} / IC_{covers}$
contains	null	null	$>\Delta_{covers} / IC_{covers}$

5. Conclusions

In this paper, we successfully used the defined area terms and relative ratio factors for metric refinements with the extension of convex hull concept to uniquely derive the eight fundamental topological relations. The outside and inside delta areas with the context of convex hull concept were defined. Consequently, we used the Outside Closeness (OC) for replacing the Expansion Closeness (EC) and the Inside Closeness (IC) for replacing the Contraction Closeness defined in other previous research. The OC alone can be used to capture *disjoint* and *meet* relations, and the combination of OC and ID measures is applied to capture *overlap* metric refinements. The IC alone can be used to characterize the contraction closeness for *inside* and *contains* with the base metric measures of *convergedBy* and *convers* relations. This paper provides a new and alternative way of metric refinements for capturing the topological relations.

6. References

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